FREE-MOLECULAR MOTION OF GAS IN CHANNEL OF FINITE LENGTH WITH SUFFICIENTLY GENERAL BOUNDARY CONDITIONS

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The problem of nonisothermal Knudsen-gas motion in a channel of finite dimensions with sufficiently general boundary conditions is considered. The particular case of gas motion in a plane finite channel is investigated.

The problem of free-molecular gas motion in a channel of finite length was first solved in [1], in which the isothermal flow of gas into a vacuum was investigated. Assuming that reflection of molecules from the channel wall conforms to the cosine law, Clausing derived an integral equation for the collision density of molecules with the wall [1]. In [2], various methods of solving the Clausing equation were considered, and diffuse—specular boundary conditions on the isothermal gas flow were taken into account [3]. In [4], expressions were obtained for the probability that molecules would enter channels of various geometry [2]. In [5], the convergence of a variational method for solving the Clausing equation was verified.

In these works isothermal free-molecular flow of a gas into a vacuum was considered. The aim of the present work is to solve the problem of nonisothermal motion of a Knudsen gas in a channel of finite length for sufficiently general boundary conditions.

Formulation of the Problem

Consider the free-molecular motion of a single-component gas in a channel of length l joining two volumes; d is the hydraulic radius of the channel [2]. The z axis of the coordinate system coincides with the channel axis. The volumes are considerably larger than the channel dimensions. The gas in the volumes is in thermodynamic equilibrium at the temperatures T_1 and T_2 and densities n_1 and n_2, respectively, i.e., the distribution function of the gas molecules in the volumes is absolutely Maxwellian

$$f_1 = n_1\left(\frac{m}{2\pi kT_1}\right)^{3/2}\exp\left(-\frac{mv^2}{2kT_1}\right), \tag{1}$$

$$f_2 = n_2\left(\frac{m}{2\pi kT_2}\right)^{3/2}\exp\left(-\frac{mv^2}{2kT_2}\right). \tag{2}$$

The distribution function of the molecules reflected from the wall, $f^+(\mathbf{v})$, is taken to be arbitrary, and approximated by a function containing a certain number of free parameters [6]. Introducing the dimensionless velocity of molecular motion $\mathbf{c}$

$$\mathbf{c} = \left(\frac{m}{2kT_1}\right)^{1/2}\mathbf{v}, \tag{3}$$

the function $f^+(\mathbf{c})$ can be expanded, by analogy with the method of [7], in powers of $\mathbf{c}$

$$f^+(\mathbf{c}) = f_0(\mathbf{v}, z)[1 + A_ic_i + Bc^2 + D_ic^2c_i + \cdots], \tag{4}$$

where $f_0(\mathbf{v}, z)$ is an absolutely Maxwellian distribution function; A_i, B, and D_i are coefficients of the expansion, depending on the coordinate z. In the present case

$$f_0(\mathbf{v}, z) = n_+(z)\left(\frac{m}{2\pi kT_+(z)}\right)^{3/2}\exp\left(-\frac{mv^2}{2kT_+(z)}\right). \tag{5}$$

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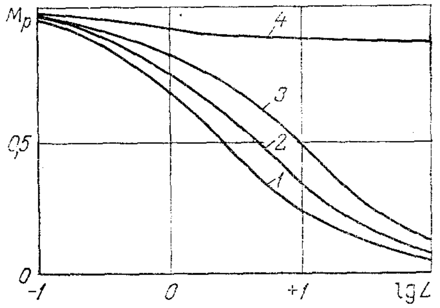

Fig. 1. Dependence of M_p on log L for different tangential-momentum accommodation factors: $\alpha = 1$ (1), 0.6 (2), 0.3 (3), 0 (4).

The density $n_+(z)$ and the coefficients in the expansion of f^+ can be found from the condition of wall impermeability and the definition of the accommodation factors of the corresponding moments of the distribution function [3, 6]. Physical considerations determine the choice of $T_+(z)$.

Obtaining the specific form of f^+, macroscopic parameters of the gas in the channel may be calculated, in particular, the reduced gas flow rate

$$G^+ = G/G_0, \tag{6}$$

where G is the mass flow rate of gas through the channel; G_0 is the gas mass flow rate from the first volume into vacuum through a hole of the same cross section as the channel

$$G_0 = \frac{mn_1}{4}\left(\frac{8kT_1}{\pi m}\right)^{1/2} S. \tag{7}$$

In the nonisothermal case, the thermomolecular pressure difference (TPD) is of interest [8]. Here a first-order steady state is observed [9], i.e., there is no mass transfer through the channel. The following formula was proposed in [10] to describe TPD

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{\gamma}, \quad \gamma = \frac{Q_T}{Q_P}, \tag{8}$$

where Q_p and Q_T are the reduced volume gas flow rates due to the pressure and temperature gradients, respectively [10, 11]. In the case of infinitely long channels, it is found that $\gamma = 1/2$ for diffuse and diffuse—specular boundary conditions in free-molecular conditions [10, 11]. However, experiments [12, 13] indicate a value of the TPD coefficient significantly different from this for various gases.

Use of Bubnov—Galerkin Method

Using the wall-impermability condition and the definitions of the Knudsen accommodation factors α_φ [3, 6], it is simple to obtain a system of K integral equations for the K terms of the expansion in Eq. (4). These equations are inhomogeneous Fredholm integral equations of the second kind, and may be solved by variational methods [2, 14]. In the limiting case $K = 1$, the Clausing equation [1, 2, 5] for a diffuse law of gas—wall interaction is obtained.

The resulting system of K integral equations will be solved by the Bubnov—Galerkin method [14]. The form of the trial functions is first chosen. In the general case, P variable constants ($P \geq K$) are introduced, and a corresponding system of P algebraic equations is obtained

$$\sum_{j=1}^{P} H_{ij} a_j = F_i, \quad i = 1, \ldots, P. \tag{9}$$

TABLE 1. Pass-Through Probability for Plane Slits

Slit No. [17]	Gas					Calc. for $\alpha = 1$
	H_2	He	Ne	Ar	Xe	
2	0,0357	0,0374	—	—	—	0,0523
3	0,0428	0,0454	0,0460	0,0418	0,0389	0,0520

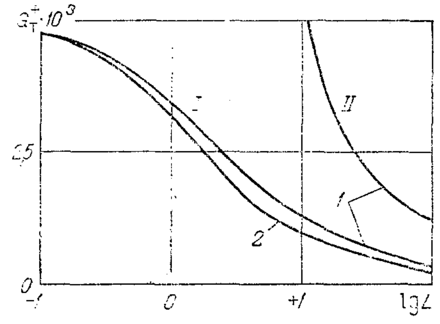

Fig. 2

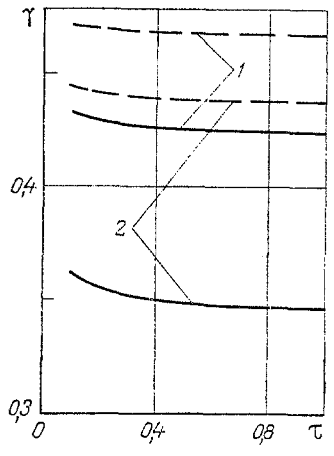

Fig. 3

Fig. 2. Dependence of G_T^+ on log L: $\alpha = 1$ (1), 0 (2); $\tau = 1.01$ (I), 1.05 (II).

Fig. 3. Dependence of TPD index on temperature ratio in volumes for channels of different length with incomplete tangential-momentum accommodations: $\alpha = 0.8$ (1), 0.6 (2); $L = 10$ (continuous curves), 1 (dashed curves).

The quantities H_{ij} depend on the accommodation factors α_φ, the reduced length of the channel $L = l/d$, and the temperature ratio of the volumes $\tau = T_2/T_1$. The free terms F_i depend on the same parameters, and are also linear functions of the molecular-density ratio for the volumes, $k_0 = n_2/n_1$. Using the solution of Eq. (9), the following expression may be written for the reduced flow rate G^+

$$G^+ = I_0 - k_0\sqrt{\tau} + I_1 a_1 + I_2 a_2 + \cdots + I_p a_p, \tag{10}$$

where I_j depend only on α_φ, L, and τ. Substituting the explicit form of a_j into Eq. (10), and performing simple transformations, the following formula is obtained for G^+

$$G^+ = N - Mk_0\sqrt{\tau}, \tag{11}$$

where

$$M = M(\alpha_\varphi,\ L,\ \tau),\ N = N(\alpha_\varphi,\ L,\ \tau). \tag{12}$$

From Eq. (11), an equation to describe TPD is obtained

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{1/2}\frac{M}{N}. \tag{13}$$

Comparison with Eq. (8) yields an expression for the TPD coefficient γ in free-molecular conditions in a channel of finite length

$$\gamma = \frac{1}{2} - \frac{\ln(M/N)}{\ln(\tau)}. \tag{14}$$

Using diffuse boundary conditions, Eq. (14) gives $\gamma = 1/2$. The same result is obtained for the diffuse—specular scheme of boundary conditions of [2]. In contrast to the diffuse and diffuse—specular models of gas—surface interaction, the expansion in Eq. (4) gives a value of less than 1/2 for the TPD coefficient, while Eq. (13) coincides in structure with the empirical formula proposed to describe the TPD experiments in [12].

Gas Motion in a Plane Channel

As an illustration of the application of the present boundary-condition model, consider gas motion in a channel proved by two infinitely wide plane plates of length l. In this case the hydraulic radius d of the channel is equal to the distance between the plates [2].

$$f^{+} = f_0(1 + A_z(z)c_z), \tag{15}$$

i.e., the accommodation of the tangential momentum at the wall is assumed to be incomplete. In this case, K = 2, and a system of two integral equations is obtained

$$n(z) = \left(1 - \frac{z}{\sqrt{1+z^2}}\right) + k_0\sqrt{\tau}\left(1 - \frac{L-z}{\sqrt{1+(L-z)^2}}\right) + \frac{1}{2}\int_0^L n(x)\frac{dx}{[1+(z-x)^2]^{3/2}} + \frac{2}{\pi}\int_0^L \frac{m(x)}{\sqrt{\tau_+(x)}}\frac{(z-x)\,dx}{[1+(z-x)^2]^2}, \tag{16}$$

$$m(z) = (1-\alpha)\left\{\frac{1}{1+z^2} - k_0\tau\frac{1}{1+(L-z)^2} + \int_0^L n(x)\sqrt{\tau_+(x)}\frac{(z-x)\,dx}{[1+(z-x)^2]^2} + \frac{3}{2}\int_0^L m(x)\frac{(z-x)^2dx}{[1+(z-x)^2]^{5/2}}\right\}.$$

The following notation has been adopted here

$$n(z) = \frac{2n_+(z)\sqrt{\tau_+(z)}}{n_1}; \tag{17}$$

$$m(z) = \frac{\sqrt{\pi}\,n_+(z)A_z(z)\tau_+^{3/2}(z)}{n_1}; \tag{18}$$

$$\tau_+(z) = \frac{T_+(z)}{T_1}; \tag{19}$$

and α is the tangential-momentum accommodation factor.

The expression obtained for the reduced flow rate G^+ is then

$$G^+ = \sqrt{1+L^2} - L - k_0\sqrt{\tau} + \frac{1}{2}\int_0^L n(z)\left(1 - \frac{L-z}{\sqrt{1+(L-z)^2}}\right)dz + \frac{1}{\pi}\int_0^L \frac{m(z)}{\sqrt{\tau_+(z)}}\frac{dz}{1+(L-z)^2}. \tag{20}$$

In the case of complete accommodation of the tangential momentum ($\alpha = 1$), for flow into a vacuum ($k_0 = 0$), the system in Eq. (16) yields the traditional Clausing equation, and the flow rate G^+ coincides with the probability that a gas molecule will pass through the channel [2, 5].

To ensure the correct choice of the form of functions n(z) and m(z) without checking the convergence of the Bubnov–Galerkin method, the system for isothermal flow into a vacuum in Eq. (16) was solved numerically by the Krylov–Bogolyubov method [15]. The accuracy of the calculation was no worse than 0.01% for α taking values from 0 to 1. The results show that, to good accuracy, n(z) may be regarded as linear and m(z) as constant. The coincidence of the accurate and variational solutions is no worse than 0.1%.

In all the calculations, $\tau_+(z)$ was taken to be a linear function of z. The calculations were carried out on a BÉSM-6 computer.

Discussion of Results

Consider isothermal gas motion in a channel under the action of a pressure difference. In this case, $k_0 = P_2/P_1$, and M and N in Eq. (11) depend only on L and α. Since when $k_0 = 1$ there must be no mass transfer through the channel, it is possible to write

$$G^+ = M_P(1 - k_0). \tag{21}$$

The subscript P denotes motion under the action of the pressure difference alone; M_P coincides, in physical meaning, with the probability that a gas molecule will pass through the channel [2]. The dependence of M_P on log L is shown in Fig. 1 for different values of the tangential-momentum accommodation at the walls. With decrease in α, M_P rises for any channel length, but remains less than unity when $L \neq 0$. If the set of diffuse–specular boundary conditions of [2] is used, the probability that a particle will pass through a channel of any length is unity in the case of completely specular reflection. Hence, the absence of tangential-momentum accommodation is not completely specular reflection. Of course, the diffusivity factor ε in the diffuse–specular boundary conditions cannot be interpreted as the tangential-momentum accommodation factor α. In the case of complete accommodation of the molecules, i.e., $\alpha = 1$, the results obtained agree completely with the data of [2, 5].

It is of interest to compare theory with experiment [17]. It is evident from Table 1 that the experimental probability that the molecule will pass through the channel is less than the theoretical value. This discrepancy may be attributed to the roughness of the channel walls, as a result of which there is a decrease in the probability.

Consider nonisothermal gas motion in a channel when the pressure in the system remains constant. In this case, $k_0 = 1/\tau$. The reduced gas flow rate is

$$G_T^+ = N - M\frac{1}{\sqrt{\tau}}. \tag{22}$$

The subscript T denotes motion under the action of a temperature difference alone. Considering the dependence of G_T^+ on log L shown in Fig. 2, it is evident that the gas flow rate rises with increase in the temperature drop at the ends of the channel and with decrease in the channel length. Together with α, G_T^+ decreases; the effect of incomplete tangential-momentum accommodation is larger for longer channels, i.e., larger surface with which the molecules interact. Whereas the values of $G_T^+(\alpha = 1)$ and $G_T^+(\alpha = 0)$ are practically equal for $\tau = 1.01$ and $L = 0.1$, for $L = 100$ the difference amounts to 34%.

In Fig. 3, the TPD index γ is plotted as a function of the temperature ratio τ in the volumes. Note the weak dependence of γ on τ for any α. For $\alpha = 0.8$–1, the maximum change in γ with τ is less than 1% and for $\alpha = 0$ it is less than 3%. Diffuse scattering of gas molecules on the channel is seen to correspond to maximum TPD. It is evident from Fig. 3 that γ decreases for incomplete tangential-momentum accommodation, which is in qualitative agreement with the experiments of [12].

NOTATION

k, Boltzmann constant; m, molecular mass; **v**, absolute velocity of molecular motion; f_1, f_2, gas-molecule distribution functions in volumes; $n_+(z)$, density of molecular collisions with wall at point z; $T_+(z)$, wall temperature at point z; S, channel cross-sectional area; P_1, P_2, gas pressure in first and second volumes; γ, universal TPD index.

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PHASE SLIP AND HEAT TRANSFER TO THE LIQUID IN FILM BOILING OF A CRYOGENIC LIQUID IN PISTON FLOW

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Published data are examined and results are given from a survey of experiments on phase slip and heat transfer to liquids.

It is now firmly established that heat transfer and hydraulic resistance in film boiling in a pipe are substantially dependent on the structure of the two-phase flow.

When a pipe is cooled by a cryogenic liquid in a heat-transfer device in a power plant, one usually finds [1-3] piston and dispersed modes of flow. These have been examined [1, 2] for liquid nitrogen, while dispersed flow has been examined [4] for hydrogen, nitrogen, and argon. It has been shown [1-4] that the speeds of liquid and vapor differ substantially in the dispersed mode, i.e., slip occurs for the two-phase flow. In the piston mode, it is assumed [1, 2, 5] that the phase speeds are equal, i.e., that the slip ratio is $u_v/u_l = 1$.

In most instances, the dispersed mode arises by decomposition of the piston mode in parts of the pipe fairly far from the inlet. Therefore, we assume that slip in the dispersed mode is one of the reasons for slip arising in piston mode, since sudden change in the phase velocities is unlikely when piston flow breaks up in a real process. It is necessary to assume constant slip in piston mode because the available studies [1, 2, 5] included no measurements of the volume vapor content, while the latter cannot be determined by solving a system of one-dimensional conservation equations written for each of the phases without the assumption of slip.

The difference between the mass-mean phase speeds explains why the few available measurements [6] on heat transfer to flowing hydrogen in pipes were not included in [1, 2, 5]. The results of [6] were also not incorporated in studies on dispersed flow. In [6] there were high mass flow speeds, and in most instances the flow was of piston type. One therefore concludes that the piston-flow models of [1, 2, 4] are rarely applicable, while the correct choice of slip is of primary importance in the analysis of data and in deriving general relationships.

The system of one-dimensional conservation equations for the piston model for steady-state flow takes the following form provided that the mass-average vapor temperature is equal to the saturation temperature (any superheating of the vapor is localized in a narrow layer near the wall, whose thickness is much less than the thickness of the vapor film):

$$\frac{dx}{dz} = \frac{\pi d}{Gr} \left(q_w - q_l \sqrt{1-\varphi}\right), \tag{1}$$

$$q_w = \frac{(1-x)G}{\sqrt{1-\varphi}\,\pi d} \frac{di_l}{dz} + q_e \left(i_l - i_{ls}\right), \tag{2}$$

$$T_v = T_s(p), \tag{3}$$

$$xG = \rho_l u_l \varphi F, \tag{4}$$

$$(1-x)G = \rho\ u\ (1-\varphi)F, \tag{5}$$

$$q_{vs} = (q_l + q_l)\sqrt{1-\varphi}. \tag{6}$$

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